OPEN GEOMECHANICS

Review History for “Characterization of plastic limit surface and bifurcation domain of geomaterials”

Rui Wang **2025**

Summary

The paper was sent to two Reviewers: Prof. Zhongxuan Yang, Zhejiang University (Reviewer 1) and Prof. Xiusong Shi, Shenzhen University (Reviewer 2). The two reviewers remained anonymous during the entire revision process. After the reviewing process was completed, the reviewers decided to disclose their identity. In the first round of review, both reviewers and the Editor recommended that minor revisions be made to the manuscript before it could be published. After minor revisions, the Editor decided to accept the manuscript without further modification

Review Round 1

Reviewer 1 (Zhongxuan Yang)

The author begins with Hill’s theory to provide a theoretical framework for determining the plasticity limit under drained triaxial paths. The dependence of the plasticity limit on the initial state, loading paths, and internal structure evolution is analyzed, thereby confirming its uniqueness. In contrast, the bifurcation domain limit cannot be considered unique. The paper is well-written and systematically derived. I suggest that the paper be considered for publication after the authors address the following issues:

- The proposed theoretical approach to characterize the plasticity limit starts by analyzing the case of conventional triaxial drained test (dq/dp=3, b=0), followed by stress proportional loading path (dq/dp=constant, b=0), and true triaxial loading (different principal stress coefficient, usually called b). How does the parameter b influence the plastic limit? Please elaborate further on this point.
- In Section 4, the analysis of the plasticity limit is based on simulation results for loose sand, while Section 6 discusses the bifurcation domain uniqueness using results for dense sand. How does the sand density or void ratio affect the results? Please clarify this.
- The concluded ultimate limit depends only on the given initial state; however, only the initial isotropic state is considered in this study. Fabric anisotropy is an important internal structural characteristic of geomaterials, and the corresponding internal variable should be incorporated into the constitutive models. The authors are invited to address the influence of initial fabric anisotropy on the results.

- How should the variation of the ultimate limit of the bifurcation domain be considered with respect to anisotropy evolution during the loading process? Please analyze this in detail. Additionally, how can the effects of induced anisotropy be distinguished from the effects of the loading path?
- Please define all the symbols/abbreviations that appear in the paper, such as cst in Eq. (43), cte in Eq. (47), R in Eq. (48), and R1 in Eq. (52).

Reviewer 2 (Xiusong Shi)

In this study, the stability and failure mechanism of geo-materials has been investigated based on the second order work criterion. It is a nice work with stepwise derivation and detailed explanation. See following specific comments:

- The authors focus on characterization of plastic limit surface with main application to granular materials. However, the mechanisms governing the yielding behavior of clays and granular soils are different. Whether it can provide a unified description for both clays and granular materials?

- The authors state that plasticity limit is unique which depends mainly on the initial reference configuration, regardless of the texture anisotropy. What is the definition of “initial reference configuration” for geo-material? How about other factors, e.g., cohesion in clayey soils?

- The introduction is too brief. The development history and recent progress of the stability and failures mechanisms of geo-materials should be discussed in detail.

- The concept of critical state is well recognized in the field of geomechanics, it is suggested to highlight the similarities and differences between the ultimate plasticity limit and critical state of soil?

- The physical meaning of the uniqueness of plasticity limit should be further explained.

- Experiential test data of different geo-materials (clay & granular materials) is recommended to support the conclusion of this study.

- In appendix B, the basic information for finite element simulation (e.g., boundary condition and properties of mesh elements) should be provided.

Author Response

We would like to thank the reviewers for their constructive comments, which highlighted the shortcomings and inaccuracies of our first version of this article. Please find below our answers to each question and comment made by the two reviewers of this work. The changes made to the article are highlighted in red.

Reviewer 1:

1-1: Traditionally, the ‘b’ coefficient is not introduced to test the influence of the loading path on the potential evolution of the plasticity limit with this parameter. Initially, when monotonic tests with a constant loading direction are imposed, the ‘b’ parameter only makes it possible to scan all the directions of the space of principal stresses and thus to characterize the three-dimensional shape of the plasticity limit in this 3d space.

1-2: It is historically recognized that the plasticity limit, when characterized by its value at the maximum of the stress deviator on a CD test, depends on the initial density of the sample. For example, on ‘the same’ dense or loose sand, a greater friction angle will be measured on the dense sample than on the initially loose sample. To simplify and homogenize the reading of the document, the results of figure 6 have been placed in an appendix, and a similar figure has been introduced instead for the model calibrated on dense Hostun sand. Thus, in the body of the text of the modified article, all the results are presented for the same dense sample, and the results for the loose sample are referred to in the appendix. With this model, the density of the sample affects the opening of the instability cones as well as the size of the bifurcation domain (see Figures 3 and 11, 12 for the dense sand and 18, 19 for the loose sand).

1-3: Indeed, for the illustrations presented with Darve's 8L model, we have considered an isotropic initial state. In fact, this model, developed conceptually several decades ago, was written mainly with phenomenological considerations at the REV scale. The rules governing the evolution of the various constitutive variables depend on:

- the current void ratio
- the current stress state
- memory variables such as the values of the stress tensor components during a cycle change in the loading direction under consideration.

Thus, parameters characterizing the sample's fabric anisotropy explicitly, as is done in models such as Sanisand or P2Psand, do not exist in this model. Instead, this model was designed with the idea of simulating a loading history from a virgin isotropic state to obtain a desired initial state. An a posteriori reconstruction of this tensor should be possible but is not part of the main subject of this work. Finally, the main topic of this work is to clarify which experiments or mechanical tests it is possible to carry out physically or failing that by direct numerical simulation with the DEM, to identify the plasticity limit unequivocally.

Please see modifications between lines 132-172.

1-4: First question: We have shown in this work that the limit of the bifurcation domain is very sensitive to the evolution of the internal variables of an elasto-plastic model (see section 6). Thus, if these variables describe an evolution of the fabric anisotropy, then a direct mathematical link (in theory) can be made between the evolution of this anisotropy and that of the bifurcation domain limit.

Moreover, Doanh's experimental work has qualitatively demonstrated this strong dependence, at least for very loose sands. After CD pre-loading, Doanh et al. showed that a loose sand could behave qualitatively like a dense sand in a CU+u test (the test is consolidated, drained and then sheared into a non-drain). It is emphasised that after CD pre-loading, the void index of the sands tested remains very high: more than 0.9 and the low densification of the material after this pre-loading cannot explain this radical change in behaviour. This proves the high sensitivity of the materials tested by the authors to change from 'unstable' to 'stable' behaviour with the evolution of micro-mechanical parameters not identified in their study but in any case, other than dry density.

Find modifications between lines 608-621, 659-661 and 674-690

Second question

the Stress-Force-Fabric homogenization relations tell us that it is impossible to have the same stress states for different anisotropies if we include in 'anisotropy' the mechanical anisotropy of the force distributions in addition to the geometric anisotropy of the directions normal to the contacts, for example. And it is also impossible to have and track different stress states for the same anisotropy. Find modifications between lines 151-154.

1-5: We have defined symbols and abbreviations that appears in different equations.

Reviewer 2:

2-1: The concepts developed in this work are very general and do not prejudge the type of material studied. For the general conclusions to be valid, the only assumption made is that a rate independent solid material is being studied. Furthermore, for this type of analysis to be of interest, the material must be non-elastic and non-associated. If it is associated, there is no possible development of instabilities before the ultimate plastic limit. So, there are no longer any questions about the appropriate choice of loading path to identify this limit. The proposed framework is already unified for clays and granular materials. It should be emphasized that the proposed framework does not prejudge the geometric shape of the plasticity limit surface. This results from the analysis of experimental tests carried out as proposed in this work. See changes in lines 132-137

2-2: This question, which overlaps largely with remarks 3 and 4 of reviewer B, concerns the core of this work. In absolute terms, we can see that the plasticity limit depends on parameters such as density, fabric anisotropy or any other microscopic parameter that can explain the evolution of the ‘mathematical’ or ‘macroscopic’ parameters describing the shape of the plasticity limit surface. The assumption of a conical surface makes it possible to parameterize this surface with a cohesion and a friction angle. In absolute terms, other parameters can also be used, such as the size of the principal axes of an ellipsoid, if this type of geometric shape is observed (or assumed).

Nevertheless, the experimental and DEM results that we are currently aware of tend to prove that the plasticity limit is not very sensitive to small variations in these microscopic parameters when a reference state is well defined. Despite these remarks, we have removed the term ‘uniqueness of the plasticity limit’. We talked about the sensitivity of this limit to changes in the internal state of the material. Furthermore, we would stress that this study remains within a very general framework and in no way postulates that the initial state can only be described by the initial density. This initial state can be described by any micromechanical parameter deemed relevant to describe the evolution of the geomaterial studied.

See modifications in the abstract as well as in lines 64-66, 132-172

2-3: We have developed the introduction a little more and included more recent references on stability and failure in geomaterials. See lines 35-43.

2-4: The concept of the critical state stipulates that for standard CD tests where a loading direction ‘ b ’ is constant in the deviatoric plane, there is a critical state of the material for which the void index and the intensity of the deviatoric stress remain constant during a monotonic evolution of the deviatoric strain. In model materials such as loose sands and normally consolidated clays, this state is reached at the deviatoric stress plateau, which corresponds to the limit of plasticity (maximum deviatoric stress). In model materials such as dense sands or over-consolidated clays, however, this state is reached post peak. Thus, the critical state is given for a strength value lower than that of the plasticity limit reached at the stress peak. However, in the latter case, this state is generally reached after the sample has lost its homogeneity (at least one shear band exists). But it is surprising to note that the critical state is very robust (experiments and DEM) and not very sensitive to imperfections and even to the initial void index if we characterize it in volume for loose material and within the shear band for dense material. It seems that the critical state is associated with a very specific granular configuration which is unique for a given material defined by its granulometry and which allows the material to withstand very large deformations. This last condition appears to constrain the granular configuration very strongly and make it unique. Micro-configurational variability is very high in terms of force chains and grain cycles, but macroscopically the state is maintained ‘indefinitely’. This is not the focus of this study, and we have made it clear that we are providing a procedure for unambiguously characterizing the plasticity limit, described as ‘ultimate’ in the sense that it is the maximum envelope of stress states admissible by the material for the particular paths that allow it. See changes in lines: 154-173.

2-5: We answered this core question by removing the notion of “unicity of the plasticity limit” as stated in our answer in question H2.

See changes in the abstract and in lines 64-66, 132-172, 608-621, 659-662 as well as the in conclusion (red color).

2-6: Results of experimental tests on loose sands have been added. To our knowledge, we are not aware of any experimental work using clays that would enrich the conclusions of this work. In fact, the low permeability of clays means that only undrained tests should be carried out. However, we have shown that this type of test should be avoided to clearly identify the plasticity limit. For normally consolidated clays, the material will be in an unstable state at the peak of ‘q’, which we know is reached before the ultimate plastic limit. See changes in lines 676-691 and the additional figure 13.

2-7: We have provided further details about this simulation. See changes in lines 817-821 and Table 1.

Editor Decision

At the end of Review Round Number 1, the managing Editor has decided to accept the revised version of the manuscript for publication without any further changes.